Satellite Constellation Design and Operation Analysis for Earth Observation Missions

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Abstract

The study addresses analytical and numerical approaches to design remote sensing satellite constellations for a given set of requirements. The requirements are given in terms of figures of merits (FOMs) which include revisit time, response time and percent coverage. The analytical analysis is done using ground track properties of the satellite orbit which identify the fundamental shift (S_F). This shift represents the gap in the ground track between each successive pass after one complete nodal period of the satellite. The relation between S_F and swath width (S_w) of the satellite is utilized to determine the number of satellites needed to get consecutive coverage of all points within a region of interest (ROI) on Earth. The analytical approach is used to design single-plane and multi-plane constellations to achieve certain FOMs.

The change in orbit configuration highly impact the ground track properties and FOMs. Understanding space environment disturbances is crucial for maintaining the designed constellation orbits and ensuring the required coverage performance. On the other hand, different operational modes might be requested for particular constellation mission which could require different satellites' configuration. Therefore, nonlinear control algorithms utilizing low-thrust maneuvers are used for constellation maintenance and reconfiguration problems.

The second part of this study discusses numerical analysis to evaluate and verify the performance of remote sensing satellite constellations. An Earth Coverage Analysis Tool (eCAT) based on MATLAB is developed by the Astrodynamics group in the Propulsion and Space Research Center (PSRC) at the Technology Innovation Institute (TII). The tool provides the capability of designing and analyzing satellite constellation coverage FOMs and investigating control algorithms efficiency to maintain and reconfigure constellation orbits and its relative placement.

Keywords— Satellite Constellation, Remote sensing, Ground Track, Constellation Maintenance and Reconfiguration, Low-Thrust Control Algorithms

I. INTRODUCTION

Remote sensing satellite systems have been widely used in various sectors including environmental monitoring, agriculture, security and defense. The rise in low Earth orbit (LEO) satellite constellation missions is driven by the fact that mission objectives can now be accomplished with smaller and more affordable satellites than in previous years. Different satellite constellation design methods, such as the Walker and Flower constellation patterns, have been developed and applied. However, these patterns are often constrained to specific configurations and orbital selections. For instance, the Walker constellation approach focuses exclusively on circular orbits with a symmetrical distribution of satellites [1]. Consequently, several studies proposed different approaches to design constellations based on a given set of mission requirements and objectives, without adhering to traditional constellation configurations.

Satellite constellation mission requirements are defined by figures of merits (FOMs) such as revisit time, response time, and coverage percentage. A combination of FOMs are used to assess the coverage quality of a given constellation [2]. On the other hand, the required image resolution, which determines the footprint geometry of the satellite's sensor, is a crucial factor to consider when designing a constellation. The image resolution is usually defined by National Image Interpretability Rating Scales (NIIRS) [3].

One of the approaches to design satellite constellation is to consider the ground track properties of the satellite trajectory. The ground track of the satellite can be used to understand when certain areas on Earth pass into the spacecraft's field of view. Works [4] and [5] discuss the analytical approach to design single-plane and multi-plane constellations based on the characteristics of the satellite ground track. The analysis uses the ground track properties of repeat ground track orbits (RGT) to design constellation with the required observation frequency or minimum ground track separation.

Numerical tools are used to evaluate and validate the performance of a given constellation. The tools use the point coverage model to assess the coverage properties of each point within an Earth's grid and calculate coverage FOMs for a particular satellite constellation. The assessment of the constellation is done either for global coverage or for regional coverage where the region of interest (ROI) is specified by the mission objective. System Tool Kit (STK) is one of the most common tool to design and evaluate the properties of the constellations. The tool provides a comprehensive capability to simulate constellation with different types of sensors and orbits and generate 3D and 2D visualizations of the scenarios [6]. However, commercial software developers often do not disclose the specifics of the algorithms and models used to generate results. This lack of details might make it difficult to interpret the simulation output data properly. Furthermore, the expense associated with commercial software can be significant, often necessitating multiple licenses to execute even basic scenarios.

The constellation designed using analytical approach deals with simplified orbital dynamics. However, the operation of the satellites requires to take into account the deviation of the orbital configuration due to the disturbances. The change in orbit configuration can dramatically impact the ground track properties and thus coverage FOMs. Understanding these disturbances is crucial for maintaining the designed constellation orbits and ensuring the required coverage performance. On the other hand, different operational modes might be requested for a particular constellation mission which could require different satellites' configuration. In [7] and [8], the concept of reconfigurable constellations (ReCons) is presented, enabling constellation missions to adapt to real-time requirements and enhance their commercial viability. The discussion includes the transition between the global observation mode (GOM) and regional observation mode (ROM) through impulsive maneuvers, employing optimization techniques to minimize various parameters such as propellant consumption and reconfiguration time.

To illustrate the developed method, this paper discusses the satellite ground track properties which is used to design constellations that provide consecutive coverage of the ROI with the required revisit time. As an example, the region on Earth between $-40°$ and $40°$ latitude is selected for the ROI. Repeat ground track (RGT) Sun-Synchronous orbits (SSO) are considered in the analysis to ensure fixed illumination conditions especially for optical telescopes. The limitation of optical satellites, which rely on daylight and clear weather conditions, makes Synthetic Aperture Radar (SAR) more advantageous as it can penetrate clouds and operate in darkness, providing consistent imagery regardless of weather or lighting conditions. Therefore, in this study both optical and SAR sensors are used to design the constellations.

The control algorithm for constellation maintenance is necessary due to deviations in the ground track from the required one. On the other hand, different operational modes might be requested for one constellation mission which could require different satellites' configuration. Therefore, the concept of constellation reconfiguration is considered to meet various mission requirements. Nonlinear control techniques utilizing low-thrust maneuvers are used for constellation maintenance and reconfiguration problems.

II. CONSTELLATION DESIGN APPROACH

One of the analytical approaches to design remote sensing constellations is to analyze the ground track properties of the satellite trajectory and satellite sensor's footprint geometry. The analysis provides a qualitative understanding and estimate of the constellation coverage properties such as revisit time.

A. Satellite Ground Track

Satellite ground track is a projection of satellite orbit trajectory on the Earth surface having the same unit vector $\mathbf{e}^E_{sat}$ as the satellite position vector $\mathbf{R}^E_{sat}$ given in Earth-Centered Earth-Fixed (ECEF) reference frame. Usually, 2D ground track plot is considered to represent the location of the satellite defined by a corresponding spherical coordinates or latitude ϕ, and longitude λ. An important parameter for Earth observation mission design is so called fundamental shift S_F. It represents longitudinal shift of satellite ground track after one nodal period. The equatorial fundamental interval can be found as follows

$$S_{F_{eq}} = (\omega_\oplus - \dot{\Omega}) \cdot T^n_{sat} = 2\pi \cdot \frac{1}{Q}, \tag{1}$$

where $\omega_\oplus$ is Earth self rotation angular velocity, $\dot{\Omega}$ is secular precession rate of satellite RAAN, $Q = R/D$ is orbit repeating factor describing number of satellite revolutions R within the repeating cycle D [9].

It should be noted that Eq. 1 represents fundamental interval at equator while it varies with latitude as follows

$$S_F(\phi) = S_{F_{eq}} \cdot \cos(\phi), \tag{2}$$

B. Sensor Geometry

Satellites are equipped with different types of sensors depending on the mission objectives and requirements. In this paper, two types of sensors are considered: optical camera and SAR. The footprint geometries of the sensors are defined to be used in the ground track analysis.

• *Optical Telescope*

Circular field of view can be used to represent optical telescope footprint. The telescope is chosen based on the desired image resolution, determined by the ground sampling distance (GSD), which categorizes the image according to NIIRS. Nadir-looking telescope gives the minimum GSD or the best resolution for a certain orbit radius R_s. However, usually, telescopes are steered with look angle θ_l to provide access to a larger area on the ground. The swath width S_w of an optical telescope is calculated as

$$S_w = 2\beta_c, \tag{3}$$

where β_c represents half of the Earth central angle subtended by the satellite's footprint (refer to fig. 1)

$$\beta_c = \frac{\pi}{2} - \theta_l - \arccos(\frac{R_s}{R_\oplus} sin(\theta_l)). \tag{4}$$

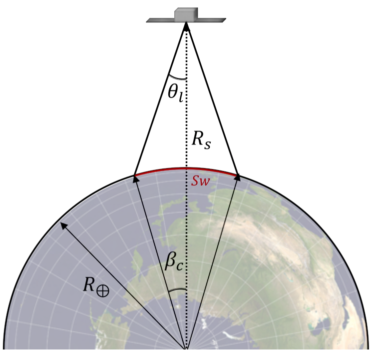

Figure 1: Optical telescope footprint geometry.

The telescope maximum θ_l defines the maximum GSD of the image. Figure 2 shows the nadir GSD for 1 m diameter optical telescope and the the look angle curves for three different GSD values.

To achieve consecutive passes over ROI, the swath width must be grater than or equal to the fundamental shift, $S_w \geq S_F \cdot \sin(i')$. For low Earth orbits (LEO) and with maximum critical look angle of $45°$, S_w is smaller than S_f (see fig. 3). Therefore, multiple satellites are needed to fill the fundamental shift gap.

• *Synthetic Aperture Radar*

SAR footprint geometry is defined by two sets of angles: look angles and exclusion angles (see fig. 4). The minimum look angle $\theta_{l_{min}}$ and maximum look angle $\theta_{l_{max}}$ define the ground range to which SAR sensor can provide coverage. The forward exclusion angle $\alpha_{forward}$ and afterward exclusion angle α_{aft} are the minimum angles between the forward and afterward projection of the velocity vector respectively and the vector to the target. SAR systems are capable of operating in different modes

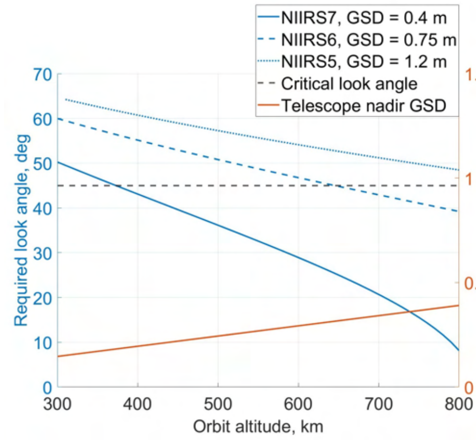

Figure 2: Nadir GSD(h) & Required look angle $\theta_l(h)$ to achieve different NIIRS classes.

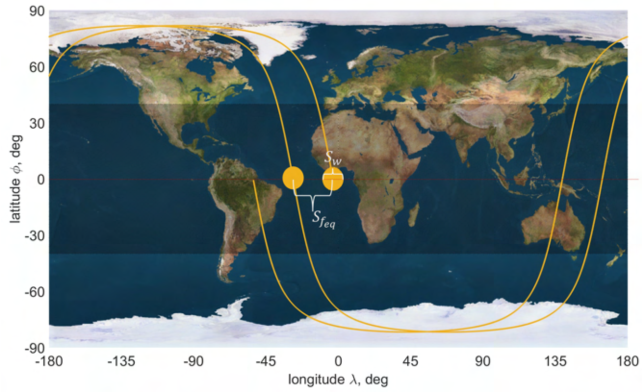

Figure 3: Fundamental shift S_F and swath width S_w at 727 km altitude.

by controlling the antenna radiation pattern [10]. One of the most used modes is the stripmap mode where the sensor has a wide swath compared to other modes of operations [10]. The swath width of the SAR sensor for a given look angle range is calculated as

$$S_w = 2(\beta_{max} - \beta_{min}), \tag{5}$$

where β_{max} and β_{min} are as follows (refer to fig. 4):

$$\beta_{max} = \frac{\pi}{2} - \theta_{l_{max}} - \arccos(\frac{R_s}{R_\oplus} sin(\theta_{l_{max}})), \tag{6}$$

$$\beta_{min} = \frac{\pi}{2} - \theta_{l_{min}} - \arccos(\frac{R_s}{R_\oplus} sin(\theta_{l_{min}})). \tag{7}$$

In Eq. 5, it is assumed that the sensor has the capability of two-side looking as followed by the geometrical restriction of the SAR field of view. It should be noted that SAR footprint contains a gap in the nadir direction, where the sensor is unable to acquire accurate measurements

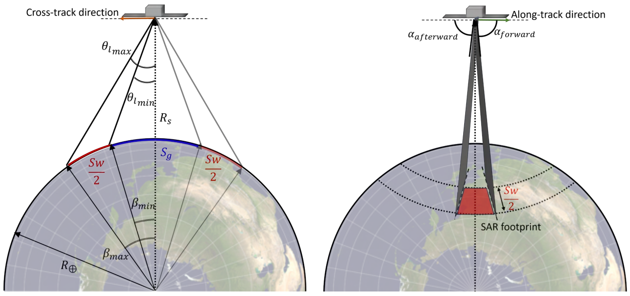

Figure 4: SAR footprint geometry.

due to technical limitations. The central gap width S_g is calculated as (see fig. 4):

$$S_g = 2\beta_{min}. \tag{8}$$

C. Constellation Design

The primary goal of the analytical constellation design method is to calculate the number of satellites required to ensure consecutive passes over the ROI with the specified revisit time.

• *Single-Plane Constellation*

The first step is to define the number of satellite in a single plane and their configuration to ensure the consecutive passes. This is done by analyzing the relation between S_F of the selected orbit and S_w of the satellite. For the selected ROI, which is the region on Earth between $-40°$ and $40°$ latitude, the maximum S_F will be at the equator, therefore $S_{F_{eq}}$ is used to calculate the required number of satellite in a single plane. The minimum number of satellites to ensure consecutive coverage of the entire ROI can be found as follows

$$n_{sats} = \left\lceil \frac{S_{F_{eq}}}{\tilde{S}_w} \right\rceil, \tag{9}$$

$$\tilde{S}_w = S_w \cdot \frac{1}{\sin(i')}, \tag{10}$$

$$i' = \arctan\left(\frac{\sin(i)}{\cos(i) - 1/Q}\right), \tag{11}$$

where i' is the apparent inclination [9].

The co-planar distribution of the optical satellites follows even distribution pattern. The mean anomaly of each satellite is calculated as follows

$$M_i = M_1 + \frac{2\pi}{n_{sats}} \cdot (i-1) \tag{12}$$

where n_{sats} is the total number of satellites in one plane, i is satellite index $i = [1 : n_{sats}]$, and M_1 is the mean anomaly of the first satellite.

SAR satellites can operate in pairs, with one satellite covering the swath gap S_g of the other, ensuring continuous coverage. This approach will work if $S_g \leq \frac{S_w}{2}$. The mean anomaly of each satellite is found as

$$M_i = \begin{cases} M_1 + \frac{\pi}{n_{sats}} \cdot (2(i-1)), & i \text{ is odd} \\ M_1 + \frac{\pi}{n_{sats}} \cdot (2(i-1)-1), & i \text{ is even} \end{cases} \tag{13}$$

• *Multi-Plane Constellation*

The second step is to calculate the number of orbital planes required to ensure the specified revisit time. The designed single-plane constellation will ensure an approximate 12 hours revisit time for all ground points within the region using SSO. This occurs because each point is covered at least twice daily—once during the satellite pass of the ascending node and another during the descending. To decrease the revisit time, a multi-plane constellation should be considered, with each plane follows the same orbital configuration. The maximum-minimum and maximum-maximum revisit time in hours of a multi-plane constellation can be estimates as:

$$T_{max-min} \approx \frac{\Omega_{min} \cdot 24\ hr}{360°} \tag{14}$$

$$T_{max-max} \approx \frac{\Omega_{max} \cdot 24\ hr}{360°} \tag{15}$$

where Ω_{min} and Ω_{max} are the minimum and maximum right ascension of the ascending node separation between the constellation orbital planes respectively and T^n_{sat} is the satellite nodal period.

III. CONSTELLATION DESIGN EXAMPLES

The ground track properties and sensor geometry are used to design optical and SAR constellations with the desired revisit time. For optical satellite, 29:2 circular RGT SSO is selected as an example. The orbit altitude is $h = 727.1$ km and inclination $i = 98.27°$. For this orbit, $S_{F_{eq}} = 24.64°$. The maximum look angle to achieve NIIRS6 images is $41.6°$ for a telescope with 1 m diameter (refer to fig. 2). The swath width of the footprint is calculated using Eq. 3 , $S_w = 12.21°$. Using Eq. 9, the required number of satellites per orbital plane is $n_{sats} = 2$. The mean anomalies of the satellites are $M_0 = 0°$, $M_1 = 180°$ using Eq. 12. The single-plane constellation ground track is shown in fig. 5.

For SAR satellites, a circular SSO orbit at altitude 600 km is selected to design the constellation. The fundamental shift for this orbit is $S_{F_{eq}} = 24.17°$. The swath width with minimum and maximum look angle of $18°$ and $45°$ respectively is $S_w = 7.42°$. Using Eq. 9, the number of satellites required for the consecutive passes is $n_{sats} = 4$ and Eq. 13 can be used to define the planer distribution

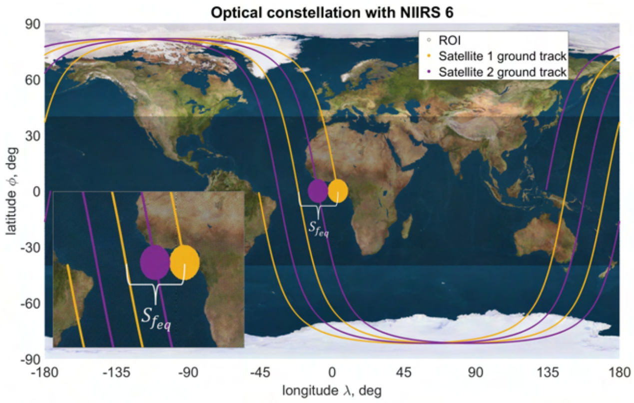

Figure 5: Optical Constellation ground Track.

of the satellites. The central gap is $S_g = 3.52°$ which is less than $\frac{S_w}{2}$, so each pair of satellites can cover footprint gaps of each other. Figure. 6 shows the single-plane constellation ground track for SAR satellites.

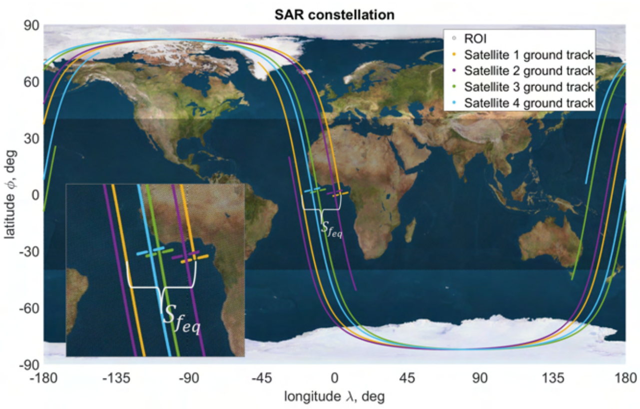

Figure 6: SAR Constellation ground Track.

IV. NUMERICAL SIMULATION

One way to verify the analytical approach and to calculate long-term coverage statistics is to perform numerical analysis. An Earth Coverage Analysis Tool (eCAT) based on MATLAB is developed to provide the required routines for the analytical approach and to numerically simulate constellation operation scenarios. In this study, the tool is used to calculate the ground track properties of the orbit and relate its fundamental interval shift S_F with satellite swath width S_w of different sensor geometries. As an example, the constellation of SAR satellites that was shown in Sec. III. is used. As shown in Sec. III., a single plane constellation will require 4 SAR satellites for the consecutive ROI coverage yielding 12 hours revisit time approximately (see fig. 7). To reduce the revisit time to 6 hrs, two planes of SAR satellites are used with 8 satellites in total. The planes are separated by 90° meaning that $\Omega_{min} = \Omega_{max} = 90°$. Therefore, $T_{max-min} = T_{max-max} = 6\ hr$. Figure. 8 shows the result of the two-planes numerical simulation which prove the analytical design approach.

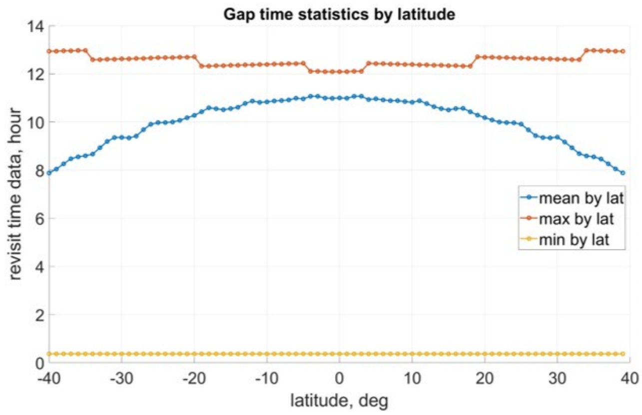

Figure 7: Single-plane SAR constellation revisit time statistics.

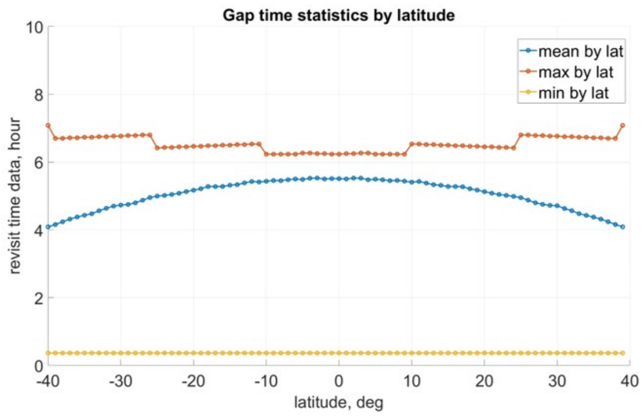

Figure 8: Multi-plane SAR constellation revisit time statistics.

V. MAINTENANCE AND RECONFIGURATION

Analytical and numerical methods used for designing satellite constellations typically rely on simplified dynamics, taking into account major disturbing force (mostly J2 effect). On the other hand, in reality, repeating ground track (RGT) orbits do not perfectly repeat their ground track after a full cycle. This is due to the fact that actual orbital motion dynamics is more complex, causing the orbit to gradually diverge from its initial configuration. In order to maintain the designed constellation configuration, the satellite orbits must be controlled using thrusters, for example. On the other hand, different operational modes and coverage properties might be requested for a particular constellation mission. Therefore, the reconfigurable constellation approach can increase the commercial value of missions by enabling the fulfillment of multiple observation needs within a particular mission.

A. *Control Algorithms*

In this paper, a Lyapunov function feedback control law based on mean orbital elements difference [11] is considered. The control algorithms are used for maintenance and reconfiguration. The error in orbital elements $\delta\text{\oe}$ is defined as the difference between current orbital elements $\text{\oe}_d$ and the desired orbital elements $\text{\oe}_{dd}$:

$$\delta\text{\oe} = \text{\oe}_d - \text{\oe}_{dd},$$

$$\text{\oe}_{dd} = \text{\oe}_c + \Delta\text{\oe}$$

where $\Delta\text{\oe}$ is the fixed set of mean orbit element difference.

For the purpose of control, the mean orbit element rate equation is approximated as:

$$\dot{\text{\oe}} \approx [A(\text{\oe})] + [B(\text{\oe})]\boldsymbol{u} \tag{16}$$

where $[A(\text{\oe})]$ matrix define the behaviour of the orbit elements under J_2 effect and control influence matrix $[B(\text{\oe})]$ is developed using Gauss' variational equations. $[A(\text{\oe})]$ and $[B(\text{\oe})]$ are well described in [11].

A positive definite Lyapunov control function based on mean orbit element tracking error $\delta\text{\oe}$ is used:

$$V(\delta\text{\oe}) = \frac{1}{2}\delta\text{\oe}^{\top}\delta\text{\oe}. \tag{17}$$

Taking the derivative of V and substituting Eq.16, the control low is derived as follows (assuming the desired relative orbits is J_2 invariant where no control is required to maintain the orbit; $[B(\text{\oe}_{dd})]$ is neglected):

$$\boldsymbol{u} = -[B(\text{\oe}_d)]\big(([A(\text{\oe}_d)] - [A(\text{\oe}_{dd})]) + [P]\delta\text{\oe}\big), \tag{18}$$

with $[P]$ being a positive definite feedback gain matrix which is a function of f and θ to make use of the fact that orbit elements are most controllable and least controllable at certain points in orbit.

B. *Orbit Maintenance Example*

Circular RGT SSO 29:2 is used as an example for orbit maintenance. A satellite with mass $m = 100$ kg and thrust $T = 10$ mN is used in the simulation. The goal of the control law is to keep the orbit semi-major axis (a), eccentricity (e) and inclination (i) within the threshold. The thresholds used in this example are $\delta_a = 100\ m$, $\delta_e = 1e-5$, and $\delta_i = 0.1°$. Figure. 9 shows how control algorithm allows satellite orbital elements within the thresholds.

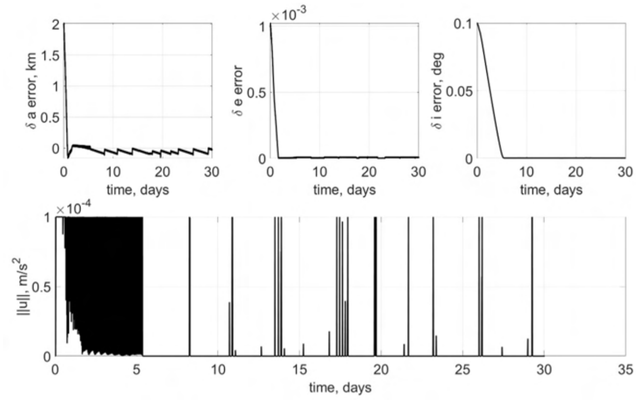

Figure 9: 29:2 RGT SSO maintenance.

C. *Orbit Reconfiguration Example*

To illustrate the constellation reconfiguration approach, an example of transferring between two different circular RGT SSO is considered. The initial orbit is 29:2 which is at altitude $h = 727.1$ km and inclination $i = 98.27°$. The target orbit is 44:3 with $h = 673.1$ km and $i = 98.05°$. The mass of the satellite $m = 100$ kg and thrust $T = 10$ mN. Figure. 10 shows the dynamics of a, e and i during the reconfiguration phase.

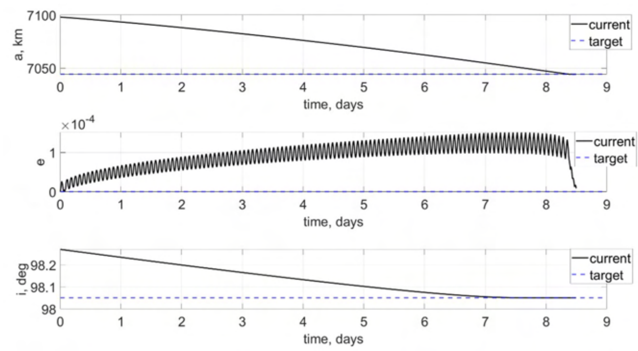

Figure 10: Reconfiguration from 29:2 RGT SSO to 44:3 RGT SSO.

VI. CONCLUSION

This paper discusses an analytical approach to design satellite constellation for remote sensing missions. The ground track properties of the satellite trajectory are used to design single-plane and multi-plane constellations with the required figures of merits (FOMs). The relation between ground track fundamental shift S_F and swath width of the sensor's footprint S_w are investigated to calculate the required number of satellite yielding consecutive coverage of a defined region of interest (ROI) in a single-plane constellation. Then, a multi-plane constellation is considered to reduce the revisit time of a single-plane constellation. A numerical simulation tool is developed to calculate the FOMs for a given constellation and verify the analytical design approach.

On the other hand, this study consider the maintenance and reconfiguration of the remote sensing constellation. A low-thrust control algorithm is used to ensure that a particular constellation will maintain its FOMs by maintaining the orbital elements of the initial configuration. In addition, the control algorithm is employed to reconfigure the constellation to satisfy different FOMs within a single mission.

The proposed approach can be utilized to design remote sensing constellations based on specific needs. This work is motivated by the announcement of the United Arab Emirates (UAE) radar satellite constellation project, Sirb. Therefore, this work can contribute to supporting the UAE's vision and long-term objectives in the space sector.

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